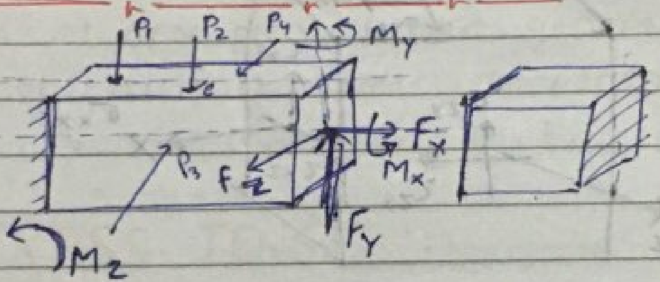
① PROPERTIES OF MATERIAL & Axial Stress

Stress develops in a body on account of restraining force or restrain deformation.

Stresses are of two types
① Normal stress → ⊥ to the section
② Shear stress → along the section

INTERNAL & EXTERNAL FORCES :-



P_1, P_2, P_3, P_4 → External Forces
$F_x, F_y, F_z, M_x, M_y, M_z$ → Internal Forces

F_x → Axial Force → Axial stress → Normal stress

F_y →
F_z → } shear force → Transvers shear stress
↳ shear stress

M_x → Torsional Movement → Torsional shear stress
↳ shear stress

M_y
M_z] → Bending moment → Bending stress → Normal stress

When the structure & loading are in same plane, it is called a 2-D condition or a planer condition. and in 2-D condition we have 3 internal forces.

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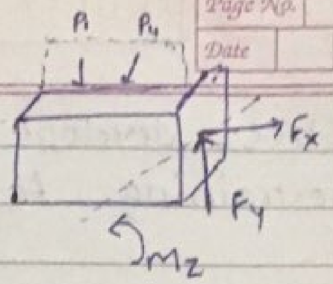
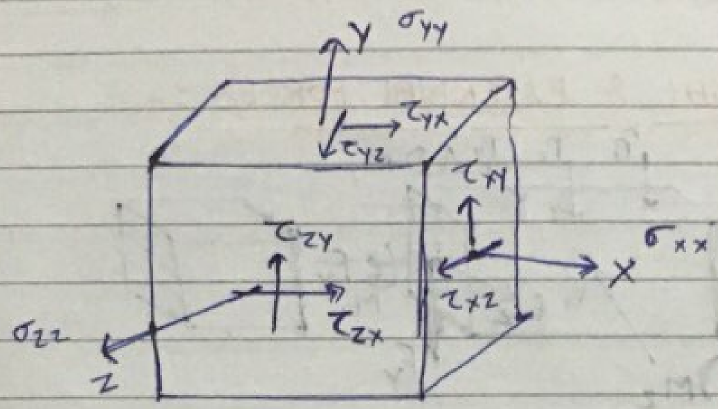
1) Axial force
2) Shear Force
3) B.M



Stresses UNDER GENERAL LOADING condition:-



at any point under General loading condition no. of stress components are 9.

Stress $\square\,\square$ — plane, Dir^n

⊕ x-plane → outward normal to the plane is in ⊕ x-dir^n

* Shear stresses on opposite faces are equal and opposite in Dir^n (It follows from force equilibirium)

* Shear stresses on adjacent perpendicular faces are equal and are directed in such a way that either both of them are pointing towards the junction or they are pointing away from the junction (this follow from moment equilibirium)

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σ_{xx}
σ_{yy}
σ_{zz}

Hence.

$$\tau_{xy} = \tau_{yx}$$
$$\tau_{yz} = \tau_{zy}$$
$$\tau_{zx} = \tau_{xz}$$

Thus at any point under general loading condition No. of distinct stress components is 6 $(\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{yz}, \tau_{zx})$

STRESS TENSION ⇒

$$\sigma = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$

→ plane

↓ dir^n.

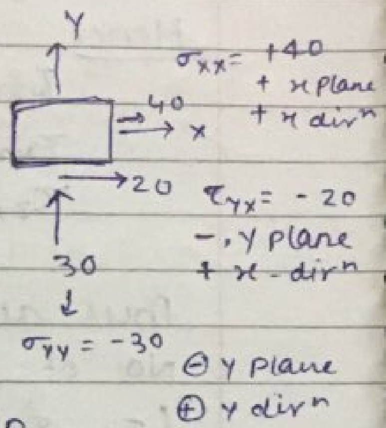
Symmetry of stress tension is on account of Moment equilibirium.

NOTE → stress, strain and MOI are 2nd order tensor

Stress is not a vector quantity because although it has magnitude and dir^n but the resultant of two stresses can not be obtained using parellelogram law of vector addition.

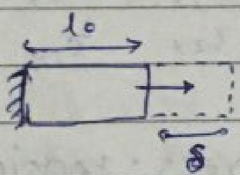
Sign convention for stresses :-

plane	dir^n	sign
+	+	= +
+	−	= −
−	+	= −
−	−	= +



NOTE For Normal stress we can have tensile stress → +ve and compressive stress → −ve

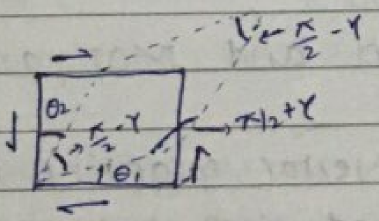
NORMAL STRAIN :-



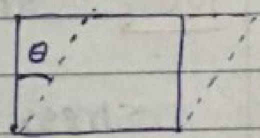
average strain.

$$\varepsilon = \text{Normal strain} = \frac{\delta}{l_o}$$

SHEAR STRAIN :-



+ve



θ = shear strain

−ve

$\theta_1 + \theta_2$ = shear strain = γ

- If angle b/w positive phases decreases shear strain will be taken as positive and if angle b/w positive phases increases, shear strain is taken as Negative.

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NOTE → positive shear stress produces +ve shear strain, -ve shear stress produces -ve shear strain.

* Normal strains can be measured using strain Gauges or extansometer. But stresses can not be measured. It can only be derived.
Hence, strain is a fundamental quantity not stress.

STRESS-STRAIN CURVE:-

This represents the charectistic property of a material.

For MILD STEEL:-

gauge length → l_0

c/s Area → A_0

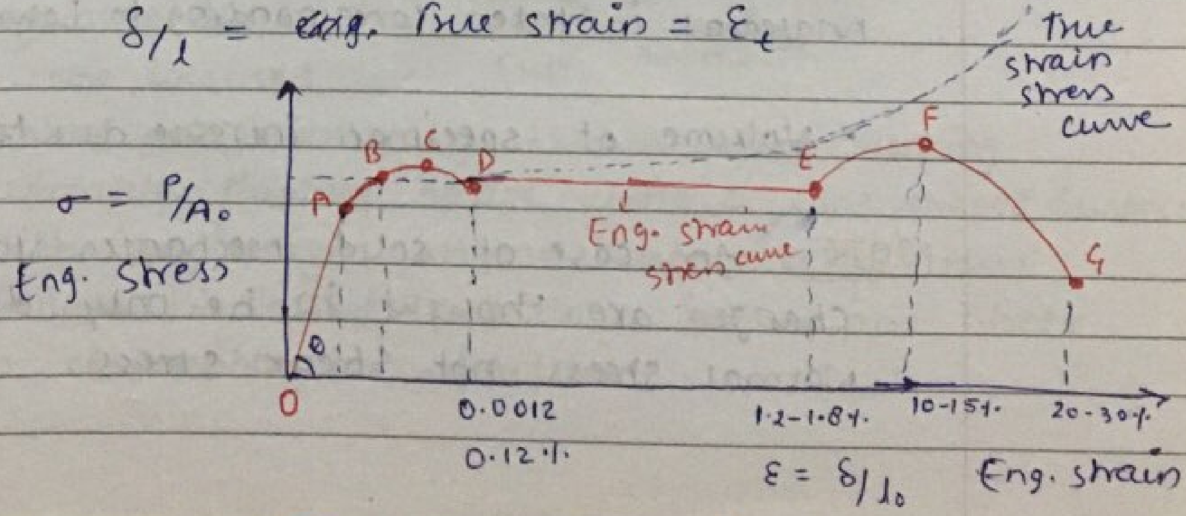
$$l_0 = 5.65\sqrt{A_0}$$

$$\sigma_{eng.} = \sigma = P/A_0$$

$$\sigma_{True} = \sigma_t = P/A$$

$$\delta/l_0 = \text{eng. strain} = \varepsilon$$

$$\delta/l = \text{true strain} = \varepsilon_t$$



A → proportional limit
B → Elastic limit
C → upper yield point
D → lower yield point
E → begining of strain Hardening
F → ultimate stress point
G → Fracture point

OA
- stress is proposnal to strain

$$\frac{\sigma}{\varepsilon} = E = \text{constant} = \tan\theta$$

- strains are very small
- volume of the speciman increases due to tension

AD
- strain starts increasing @ greater Rate
- If Material is unloaded before elastic limit B, the original shape & dimention will be regained instantaneously. That is there no residual strain.

- upper yield corresponds to transiet condition lower yield corresponds to load require to Maintain yield Hence, yield strength of Material is taken corresponding to lower yield.

- Volume of speciman increase due to tension.

NOTE In case of solid Mechanics volume changes are thought to be only due to Normal stress not shear stress

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- Upto point D Normal stress is primarly responsible for deformation.

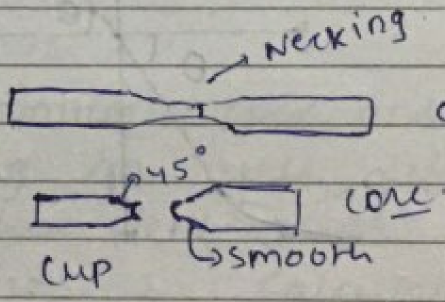
DE → plastic Zone

- Strain increases without significant increase in stress
- Deformation is caused due to slipage of material on oblique plane enhance deformation is primarily due to shear stresses Thus volume change is 'Zero'
- strains are permanent

EF → Strain Hardening region →

- Beyond point E Material starts offering resistance against deformation. This is due to change in the crystline structure of the Material.

FG → NECKing :-



① c/s of speciman begains to decrease at some localised location due to instability. This is called Necking, ultimately Rupture occurs along a cone shaped surface. With angle from original surface = 45° this failure is called cup-cone failure and shear is responsible for failure

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- percentage reduction in C/s Area upto the time of fracture point is about 50%.

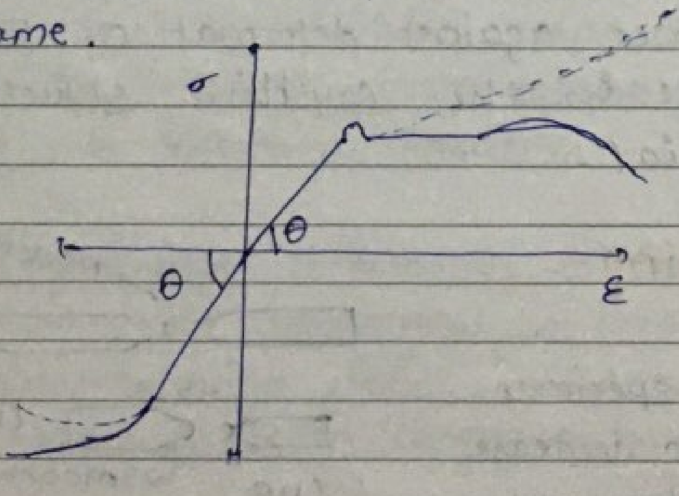
NOTE In Ductile Materials shear is responsible for fracture.

- More plastic elongation → more ductile.

MILD STEEL IN COMPRESSION:-

- The stress - strain curve in compression would be essentially same through it's initial straight line portion and through yield and strain hardening and through the begaining of yielding and strain Hardening.

- IN compression no necking occurs and Modulus of Elasticity in tension & compression are same.



- True stress in compression will be smaller.

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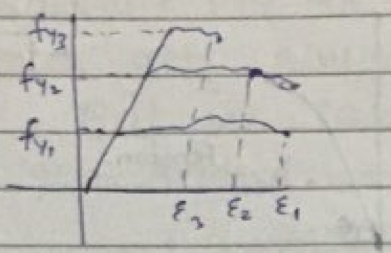
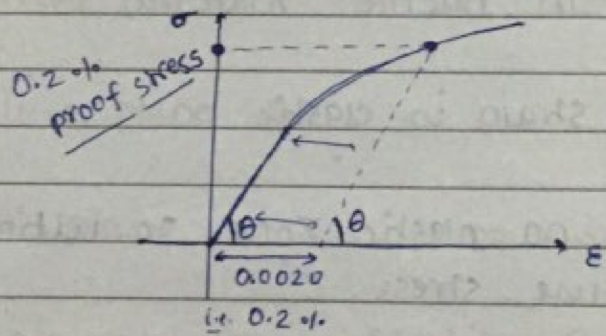
Stress - Strain curve for Other grade of STEEL :-



Strength, Ductility and corroison resistence can be altered by alloying, heat treatment and using various manufacturing processes.
But the Modulus of Elasticity for various grades are same and as the yield strength increases Ductility Reduces.

For Aluminium $ COPPER :-



In case of Aluminium copper and other materials having no well defined yield point, the yield stress for calculation purpose is calculated using offset method, in which we start with 0.0020 (0.2 %) strain and go parallel to initial straight line portion of the σ - ε curve. The point where the line intersects the σ - ε curve corresponds to 0.2 % proof stress. which is taken as yield stress for calculation purpose.

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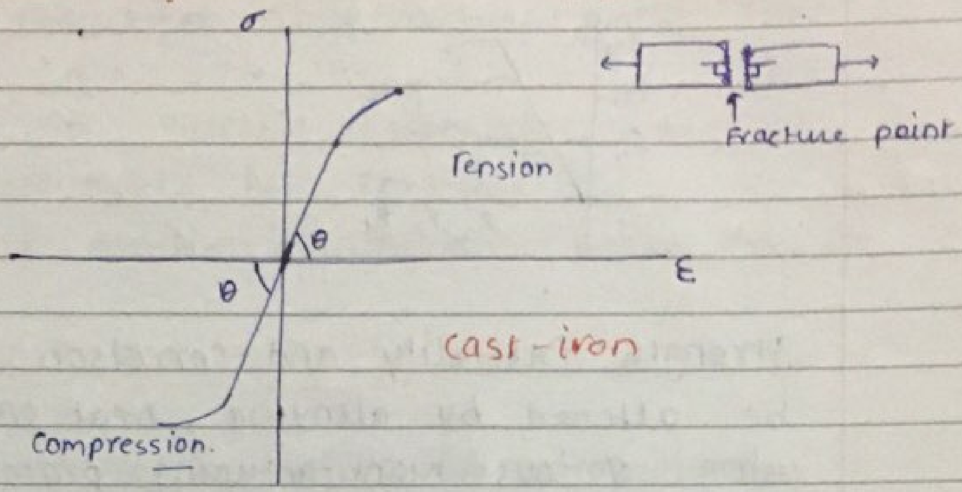
Stress-strain curve for Brittle Material:-



- The linear elastic range in tension is smaller than in compression

- Strain at rupture is very small as compare to that in Ductile material

- Rupture strain is elastic

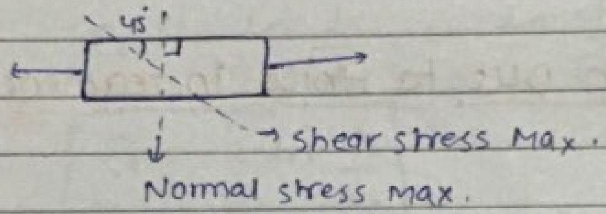
- There is no plastic zone so ultimate stress is Rupture stress.

$$\text{permissible stress} = \frac{\text{Rupture stress}}{F.O.S}$$

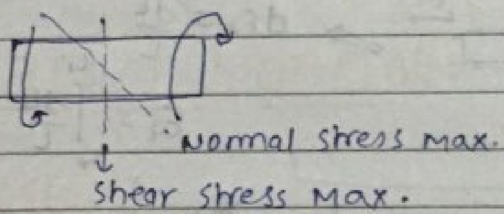
- No necking occurs

- Modulus of Elasticity in tension & compression are same
- Normal stress is responsible for fracture
- fracture plane will be granular
- failure plane is 90°

- Due to presence of Microscopic cracks in the structure of the material. During Tension. The crack gets elongated and early fracture occurs, But in case of compression the cracks get Bridged.



Tension

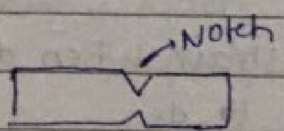


Torsion

	Ductile (fracture due to shear)	Brittle (fracture due to Normal)
Tension →	failure plane 45°	90° plane
Torsion →	90° plane	45° plane

* • Material which is Ductile @ Normal Temp. can get fractured at very small strain under sub zero temp. (less temp.) Thus it behaves as Brittle Material. This type of fracture is called Brittle fracture

* • presence of Notch can also lead to Brittle fracture



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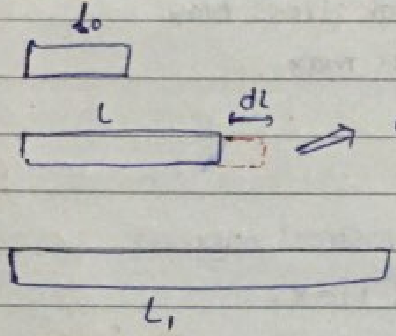
NOTE Brittleness is not the Absolute property of Material. It depends upon temprature, Stress condition, the way of loading Application.

Behaviour of Material under Static loading and Dynamic loading would be Different

True strain Due to Finite increament of loading ⇒



$$\Rightarrow d\varepsilon_t = \frac{dL}{L}$$

$$\varepsilon_t = \int_{L_0}^{L_1} \frac{dL}{L}$$

$$* \boxed{\varepsilon_t = \ln \frac{L_1}{L_0}}$$

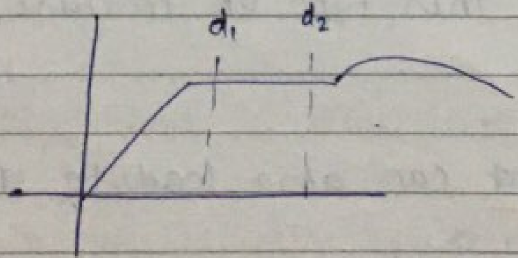
$$L_1 - L_0 = \delta$$

$$\varepsilon_t = \ln\left(\frac{L_0 + \delta}{L_0}\right)$$

$$\varepsilon_t = \ln(1 + \delta/L_0)$$

$$\boxed{\varepsilon_t = \ln(1+\varepsilon)}$$ → Engineering strain

↓ True strain



Find true strain when dia in plastic zone changes from d_1 to d_2

Since No volume changes under plastic zone

$$\frac{\pi d_1^2}{4} l_1 = \frac{\pi}{4} d_2^2 l_2$$

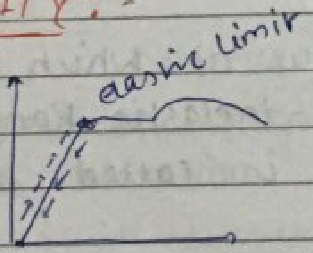
$$\frac{l_2}{l_1} = \frac{d_1^2}{d_2^2}$$

$$\varepsilon_t = \ln \frac{l_2}{l_1} = \ln \frac{d_1^2}{d_2^2}$$

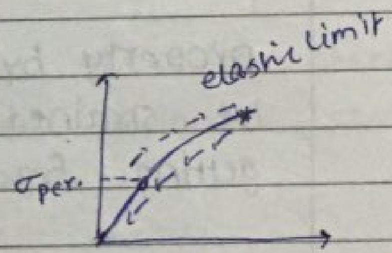
* $$\boxed{\varepsilon_t = 2 \ln \frac{d_1}{d_2}}$$

PROPERTIES OF MATERIAL :-

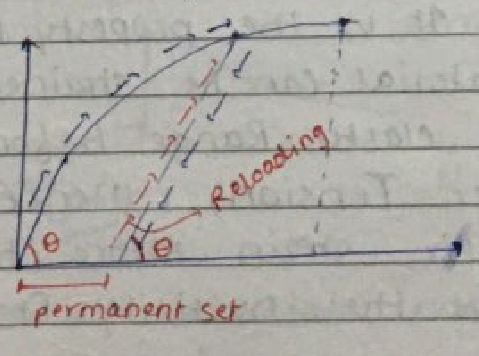
① ELASTICITY :-



Linearly Elastic



Non-Linearly Elastic



- It is the property by virtue of it the material regains it's original shape and dimension instanteneously, when unloaded before elastic Limit.

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- If Material is loaded beyond elastic limit and then unloaded, the Material ~~regai~~ has a permanent set and unloading curve will be parellel to initial straight line portion of the stress-strain curve.

On Reloading the stress strain curve will follow the unloading curve and hence proportional limit gets extended But Ductility gets Reduced

PLASTICITY:-

property by virtue of which a material can be strained into inelastic Range without getting fractured is called plasticity.

DUCTILITY:-

It is the property by virtue of which Material can be strained ~~into~~ significantly beyond the elastic Range before Fracture (under Tension). Hence greater is the inelastic strain before fracture greater is the Ductility. It is related to tension.

Malleability: It is related to compression and it is the property due to which Material can be deformed or stressed in different dir^n under Compression (rolling, pressing, hammering)

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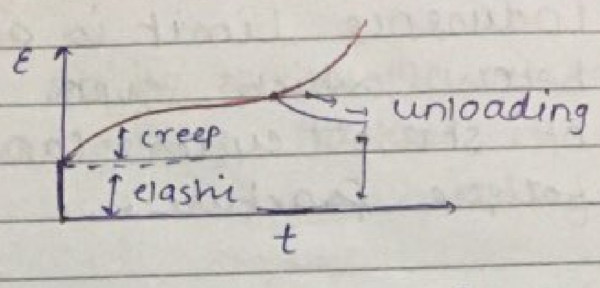
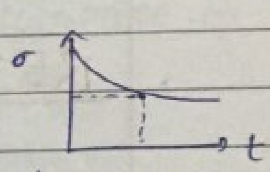
CREEP:-



- It is the property due to which Material undergoes additional deformation (over and above elastic deformation) with passes of time under sustained loading within elastic limit.

- Creep strain is a plastic strain. it depends upon temp. and stress levels

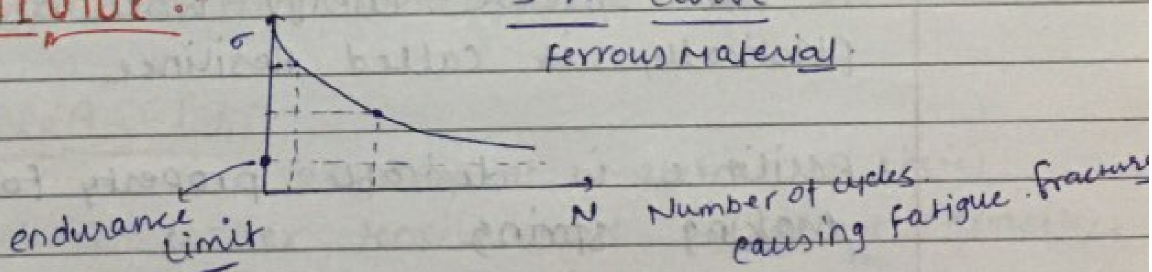
RELAXATION:-



Decrease in a stress in steel as a result of creep under prolonged strain is called relaxation.

FATIGUE:-

S-N curve ·
Ferrous Material



- Detoriation of Material under repeated cycles of loading resulting in progressive cracking that ultimatly produces fracture is called Fatigue
- Load causing fatigue failure is much smaller than load that can be sustained statically.
- Fatigue fracture depends upon Magnitude of loading and No. of cycles of loading.

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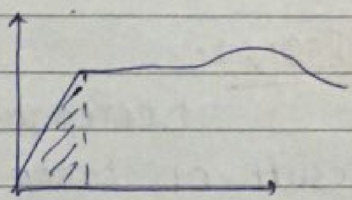
- Indurence limit is a stress level below which even infinite Number of stress cycles can not cause fatigue Fracture.

- For structural steel Indurence limit is approx 50% of ultimate strength.

- Corroison, stress level, surface finish, stress concentration etc. affects the fatigue strength.

RESILINCE :-



property of Material due to Which it can observe energy when deforms plastically is called Resilince

- Resilince is desirable property for making spring

NOTE Area under stress-strain curve = strain energy absorved per unit volume of material.

- Area under stress-strain curve upto elastic limit is called Modulus of Resilience
Greater is the Modulus of Resilienc better is the material for Making spring

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TOUGHNESS :-

Ability to absorve Mechanical energy without getting fracture is called Toughness

- Area under stress-strain curve upto Fracture is called Modulus of toughness.

- Toughness is a desirable property against impact loading.

- Mild steel is more tough as compare to cast Iron.

HARDNESS :-

Ability to resist scratch or abration is called hardness.

- larger is the yield stress harder is the material.

TENACITY :-

property of Material to resist fracture under tensile load is called Tenacity.

VISCO ELASTIC MATERIAL :-

Material having both viscous and elastic property is called viscoelastic and in this case we'll have time dependent stress-strain curve.

$$\sigma \propto \varepsilon \rightarrow \text{solid}$$

$$\tau \propto \frac{dx}{dt} \rightarrow \text{Fluid}$$

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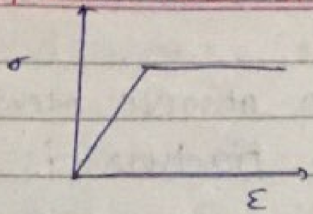
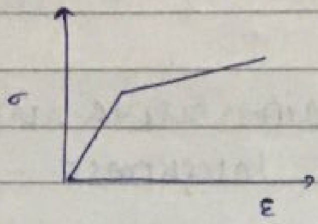
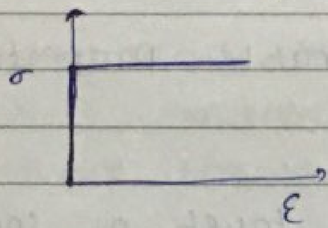
Approximate stress-strain curves



Elasto - plastic

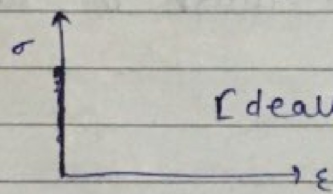


Elasto - plastic with strain Hardening.

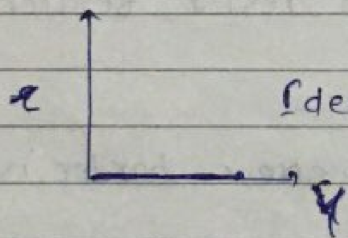


Rigid plastic

ex. Toothpaste



Ideally rigid



Ideal fluid behaviour

AXIAL STRESSES :-

HOOKE'S LAW :-

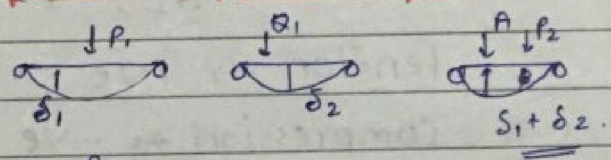
$\sigma \propto \varepsilon$

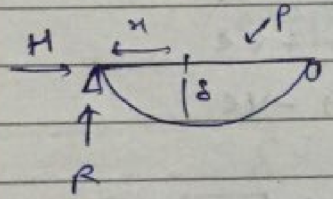
$\boxed{\sigma = E\varepsilon}$ → Modulus of Elasticity.

As per Hooks law for Homogeneous & Isotropic Material stress ∝ strain. for linearly elastic Material.

Hooke's law is valid till proportional limit

principle of superposition :-





$M_A = Rx + H\delta$ → 2nd order analysis

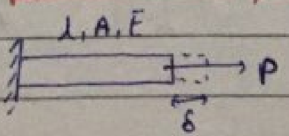
$M_A = Rx$ → 1st order analysis

According to this principle each of the loading produces it's effect independent of other and total effect is the summesion of effect due to individual forces.

For the validity of principle of superposition Material Response should be

1) Linealy Elastic (Hooke's law valid)
2) Deformations should be small.

DEFORMATION OF MEMBER UNDER AXIAL LOAD :-



$\sigma = P/A$

$\varepsilon = \dfrac{\delta}{l}$, $\sigma = E \cdot \varepsilon$

$\dfrac{P}{A} = E \cdot \dfrac{\delta}{l} \Rightarrow \boxed{\delta = \dfrac{Pl}{AE}}$

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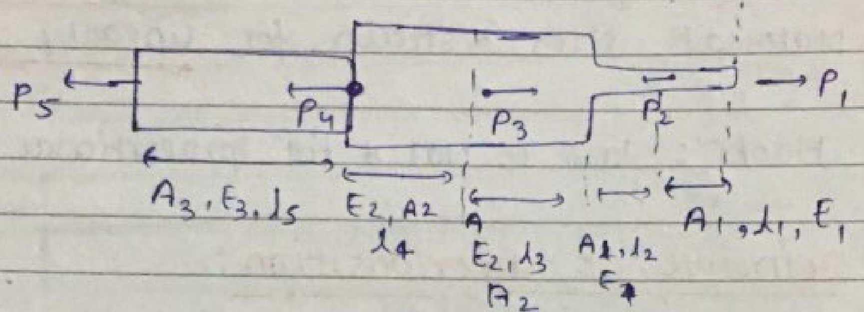
*

$$\frac{P}{\delta} = \text{Axial stiffness} = \frac{AE}{l} \rightarrow \text{Axial Rigidity.}$$

STAPPED.

AXIAL DEFORMATION IN STAPPEDBAR :-

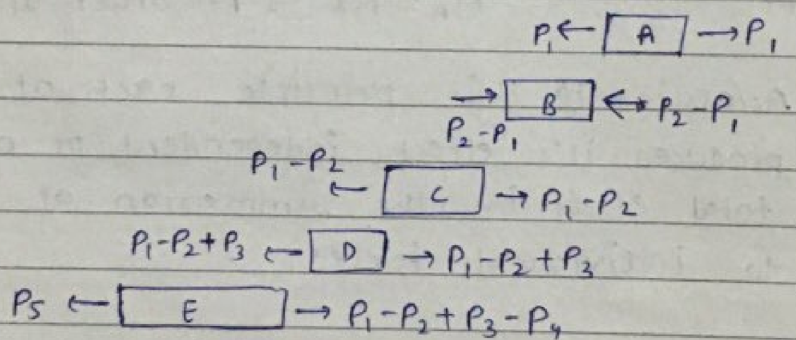


Tension → +ve

Compression → −ve

Elongation → +ve

Contraction → −ve



$$\delta_{total} = \delta_A + \delta_B + \delta_C + \delta_D + \delta_E$$

$$= \frac{P_1 l_1}{A_1 E_1} - \frac{(P_2 - P_1) l_2}{A_1 E_1} + \frac{(P_1 - P_2) l_3}{A_2 E_2}$$

$$+ \frac{(P_1 - P_2 + P_3) l_4}{A_2 E_2} + \frac{(P_1 - P_2 + P_3 - P_4) l_5}{A_3 E_3}$$

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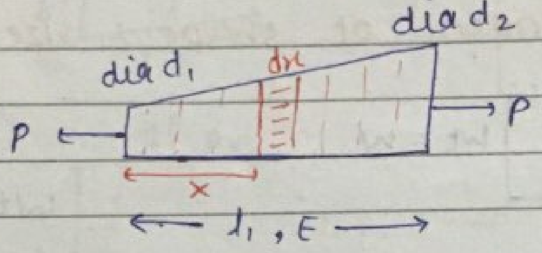
NOTE :- The BAR should be split into components in such a way that for the complete length of each individual component E, P & A should be constant.

TAPERED BAR :-

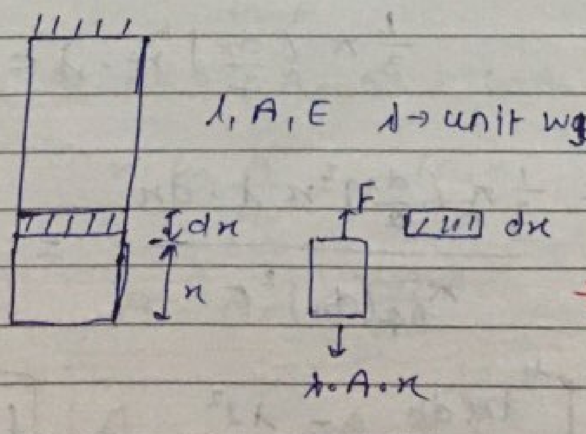


$$P \leftarrow \boxed{\;} \rightarrow dx \qquad \text{area} = \frac{\pi}{4}\left(d_1 + \frac{d_2-d_1}{l}x\right)^2$$

$$\delta = \int d\delta = \int_0^l \frac{P\cdot dx}{\pi/4\left[d_1 + \frac{d_2-d_1}{l}x\right]^2 E}$$

*
$$\boxed{\delta = \frac{Pl}{\pi/4\, d_1 d_2 E}}$$

Elongation due to self weight :-

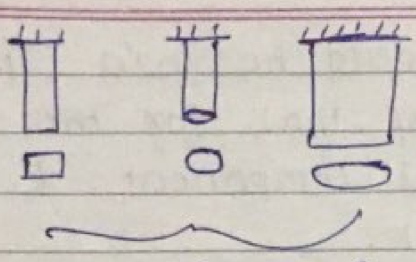


$l, A, E \quad \lambda \rightarrow$ unit wg.

$F = (\lambda A)x$

$$\delta = \int d\delta = \int_0^l \frac{\lambda A x\, dx}{AE}$$

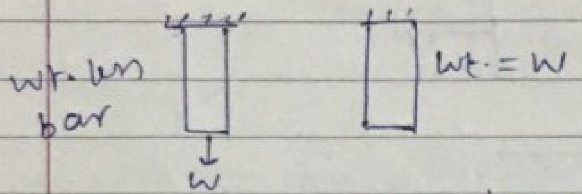
*
$$\boxed{\delta = \frac{\lambda l^2}{AE}}$$



λ, A, E

same δ

NOTE : Elongation due to self weight is independent of shape & size of c/s



$Wt. = W$

$W = \lambda A l$

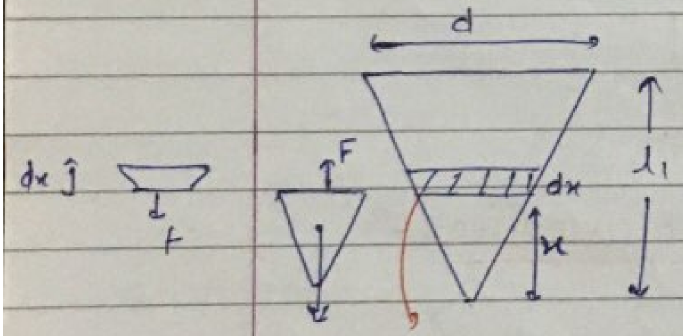
$\delta = \frac{W}{Al} \cdot \frac{l^2}{2E}$

elongation $= \delta_2$

elongation $\to \delta_1$ due to self wt.

$$\boxed{\delta_1 = \frac{\delta_2}{2}} \ *$$

$$\delta = \frac{Wl}{2AE}$$



conical

l_1

$E, \lambda \to$ unit wt.

$d_x = \frac{d}{l} x$

$$\frac{1}{3}\pi \left(\frac{d_x}{2}\right)^2 x \cdot \lambda = F$$

$$d\delta = \frac{\frac{1}{3}\pi \left(\frac{d_x}{2}\right)^2 x \lambda \cdot dx}{\frac{\pi}{4}(d_x)^2 E} = \frac{\lambda \cdot x\, dx}{3E}$$

$$\delta = \int_0^l \frac{\lambda x\, dx}{3E} = \frac{\lambda l^2}{6E} \to \boxed{\delta = \frac{\lambda l^2}{6E}} \ *$$

Independent of Diameter

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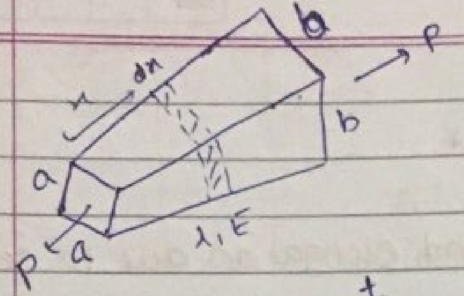
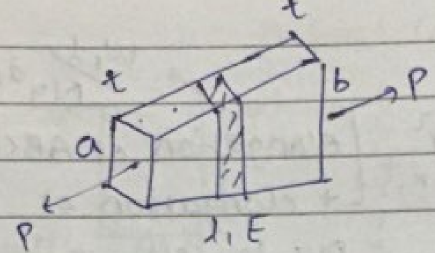
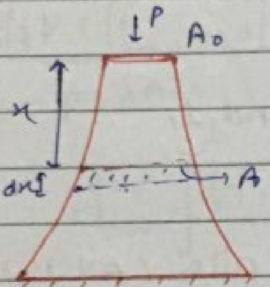


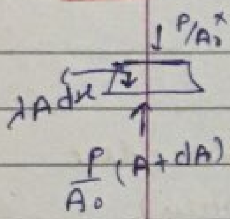
$$\boxed{\delta = \frac{P l}{a b E}}$$



$$\boxed{\delta = \frac{P l \, \ln(b/a)}{(b-a) t E}}$$

Que





λ, E, λ → unit wt.

x-sec varies in such a way that stress at every x-sec is same (ie bar of constant strength) Find the area of x-sec at Bottom)

$$\frac{P}{A_0} \times A + \lambda A dx = \frac{P}{A_0}(A + dA)$$

$$\int \frac{dA}{A} = \frac{\lambda}{(P/A_0)} \int dx$$

$$\ln A = \frac{\lambda A_0 x}{P} + c$$

at $x = 0$, $A = A_0$, $\ln A_0 = c$

$$\Rightarrow \ln \frac{A}{A_0} = \frac{\lambda A_0 x}{P}$$

$$A = A_0 \cdot e^{\frac{\lambda A_0 x}{P}}$$

$$\boxed{A_{bottom} = A_0 \, e^{\frac{\lambda A_0 l}{P}}}$$

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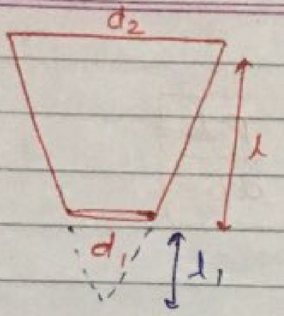


l, E, λ

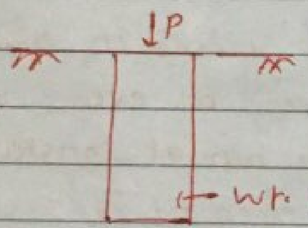
Find elongation due to self wt.

$$\frac{\lambda (l+l_1)^2}{6E} = \frac{\lambda l_1^2}{6E} + \Big[\text{elongation of ABCD due to } W_1 \nearrow \frac{W_1 l}{\pi/4\, d_1 d_2 E}$$
$$+ \text{ elongation of ABCD due to self wt.}\Big]$$

↓ find.

$$\frac{d_2}{l+l_1} = \frac{d_1}{l_1} \rightarrow \text{find } l_1$$

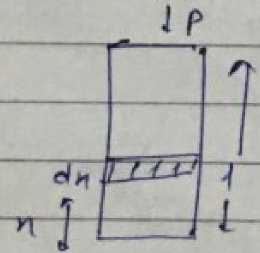
Que.

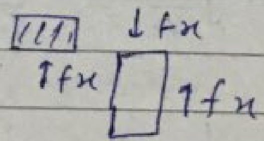


friction pile (All resistance due to friction only)

• frictional force / unit length = f

Find compression in pile.





$$P = fl$$

$$\delta = \int_0^l \frac{fx\, dx}{AE} = \frac{fl^2}{2AE}$$

$$\boxed{\delta = \frac{Pl}{2AE}}$$

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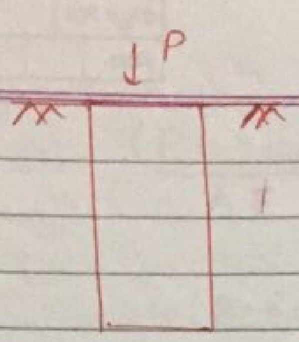
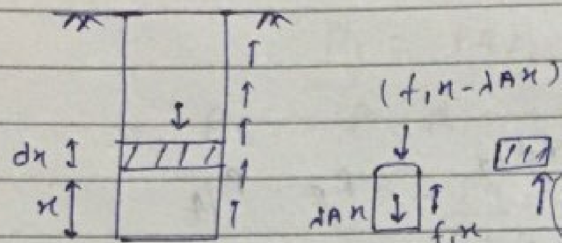


λ, E, A, l
Friction pile
frictional force per unit length $= f_1$
Find comp. in the pile ?



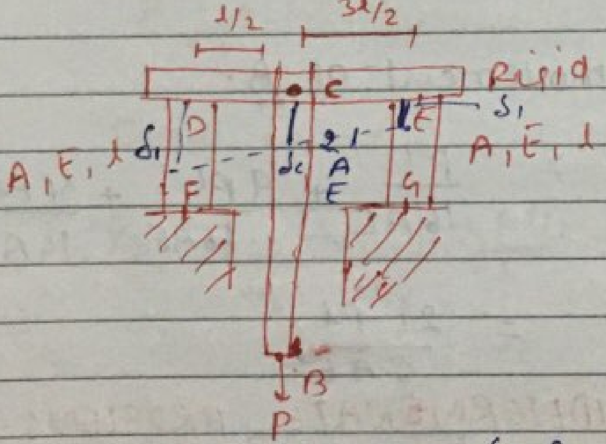
$(f_1x - \lambda Ax) \rightarrow \frac{Px}{l}$

$f_1 l = P + \lambda A l$

$$\delta = \int_0^l \frac{(f_1 x - \lambda A x)\,dx}{AE}$$

$$\delta = \frac{(f_1 - \lambda A) l^2}{2AE} = \frac{Pl}{2AE}$$

Que



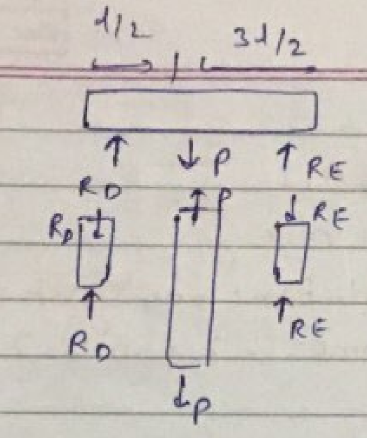
Find distance of Movement of B

$$\delta_c = \delta_2 + \frac{\delta_1 - \delta_2}{2l} \times \frac{3l}{2}$$

Downward ~~moment~~ movement of B $= \delta_c +$ elongation of BC

$$= \delta_2 - \frac{3\delta_2}{4} + \frac{3\delta_1}{4} + \Delta_{BC}$$

$$= \frac{\delta_2}{4} + \frac{3\delta_1}{4} + \Delta_{BC}$$



$R_D + R_E = P$

$R_E \times 2l = P \times l/2 \qquad R_E = P/4$

$R_D = 3P/4$

$\delta_1 = 3Pl/4AE$

$\delta_2 = Pl/4AE$

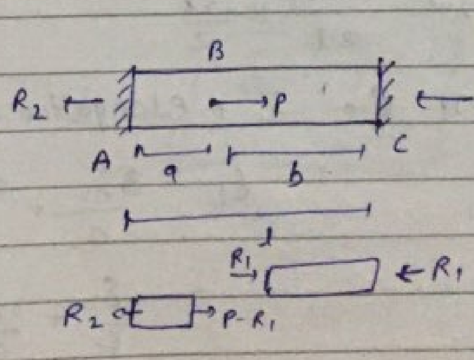
$$\Delta_{BC} = \frac{P(2l)}{AE}$$

downward movement of $\underline{B}$.

$$= \frac{Pl}{16AE} + \frac{9Pl}{16AE} + \frac{31Pl}{16AE}$$

$$= \frac{21Pl}{8AE}$$

STATICALLY INDETERMINATE PROBLEM:-



$\boxed{\delta_{AB} + \delta_{BC} = 0}$ compatibility $\underline{eq^n}$.

$$\frac{(P-R_1)a}{AE} + \frac{(-R_1)b}{AE} = 0$$

$$(P-R_1)a = R_1 b$$

$$R_1 = Pa/a+b$$

$$R_1 = Pa/l$$

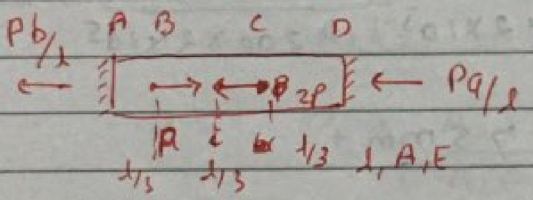
$$R_2 = P - R_1$$

$$= P - \frac{Pa}{a+b}$$

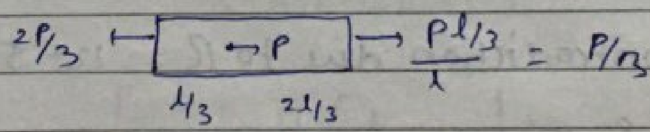
$$= \frac{Pb}{a+b}$$

$$R_2 = \frac{Pb}{l}$$

Que.



Find R_A, R_D



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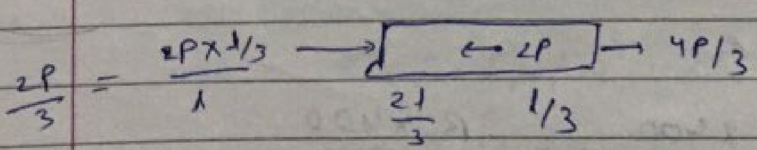

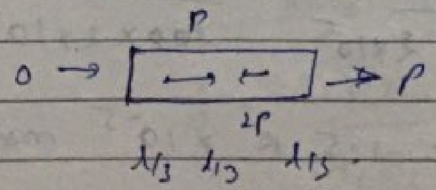

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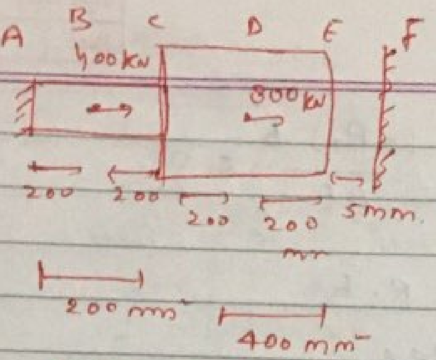
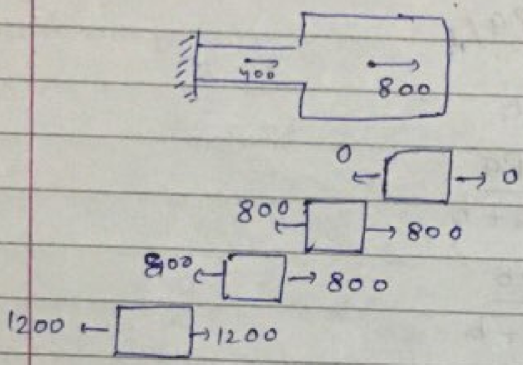


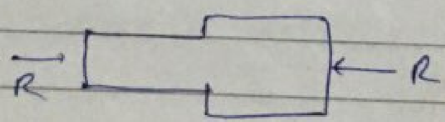
$E = 2\times10^5\ N/mm^2$

Find reaⁿ @ F ?

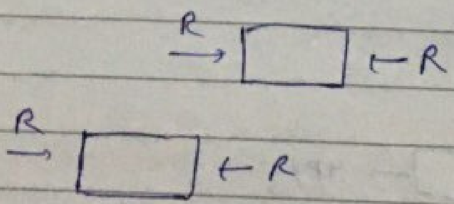


$$\delta_{free} = 0 + \frac{800\times10^3\times200}{400\times2\times10^5} + \frac{800\times10^3\times200}{200\times2\times10^5} + \frac{1200\times10^3\times200}{200\times2\times10^5}$$

δ_{free} = 12 mm ~~7 5~~ mm.



Contradiction due to $R = 12 - 5 = 7$ mm.



$$\delta = \frac{R\times400}{400\times2\times10^5} + \frac{R\times400}{200\times2\times10^5}$$

$7\ mm = 1.5\ R\times10^{-5}\ mm$

$R = 466.67\ kN$

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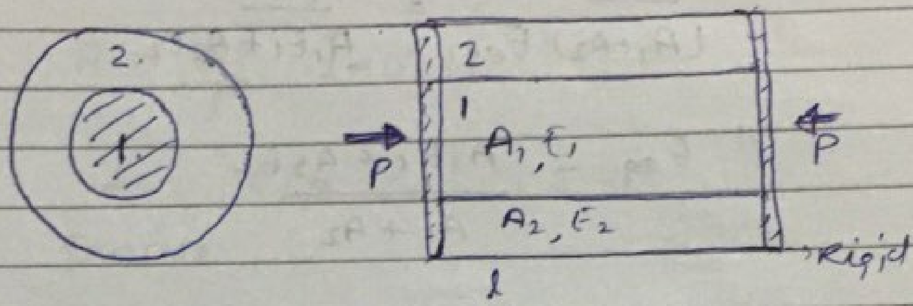
COMPOSITE BAR :-

A BAR composed of two diff^nt Material having equal change in length in both the material is said to be a composite BAR



$$P = P_1 + P_2$$

$$\frac{P_1 l}{A_1 E_1} = \frac{P_2 l}{A_2 E_2}$$

$$\frac{P_1}{P_2} = \frac{A_1 E_1}{A_2 E_2}$$

$$P = P_1 + \frac{A_2 E_2}{A_1 E_1} \cdot P_1$$

$$P_1 = \left(\frac{A_1 E_1}{A_1 E_1 + A_2 E_2} \right) P$$

$$P_2 = \left(\frac{A_2 E_2}{A_1 E_1 + A_2 E_2} \right) P$$

$$\sigma_1 = P_1 / A_1$$

$$\sigma_2 = P_2 / A_2$$

$$\delta = \frac{P l}{A_1 E_1 + A_2 E_2}$$

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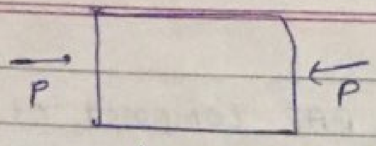


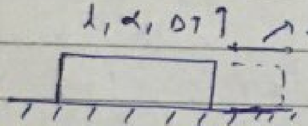
$l, (A_1 + A_2), E_{eq.}$

$$\delta = \frac{Pl}{(A_1 + A_2) E_{eq.}} = \frac{Pl}{A_1 E_1 + A_2 E_2}$$

$$\Rightarrow E_{eq.} = \frac{A_1 E_1 + A_2 E_2}{A_1 + A_2}$$

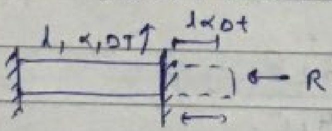
TEMPRATURE STRESSES :-

Case-1.



Frictionless

stress developed = 0



$$\frac{R(l)}{AE}$$

$$l \alpha \Delta T = \frac{Rl}{AE}$$

$$R = \frac{AE}{l} (l \alpha \Delta T)$$

$\hookrightarrow$ Deformation prevented

$$\sigma = \frac{R}{A} = E \alpha \Delta T$$

$\downarrow$ compressive,

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case-2.

$l, \alpha, \Delta T \downarrow$

$l\alpha\Delta T$

$\downarrow$

$\frac{Rl}{AE}$

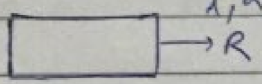
$$R = \frac{AE}{l}(l\alpha\Delta T)$$

$$\sigma = R/A$$

Tensile

NOTE

$l, \alpha, \Delta T \uparrow$



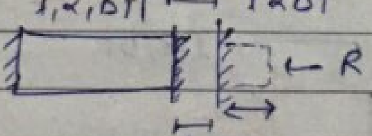
Total change in length = change in length Due to temp + change in length Due to loading.

$$0 = l\alpha\Delta T + \frac{Rl}{AE}$$

$$R = -\frac{AE}{l}(l\alpha\Delta T)$$

support yield case

$l, \alpha, \Delta T \uparrow$ $\quad l\alpha\Delta T$



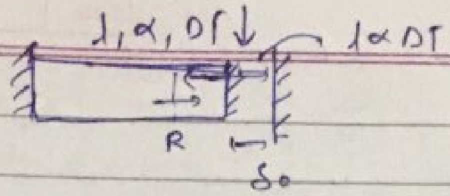
$\frac{Rl}{AE}$

$$l\alpha\Delta T - \frac{Rl}{AE} = \delta_0$$

$$R = \frac{AE}{l}(\underbrace{l\alpha\Delta T - \delta_0}_{\text{deformation prevented}})$$

$$\sigma = R/A$$

compression

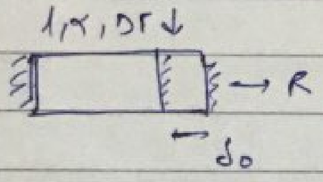


$$\frac{Rl}{AE}$$

$$R = \frac{AE}{l}(l\alpha\Delta T - \delta_0)$$

$$\sigma = R/A$$

Tensile.

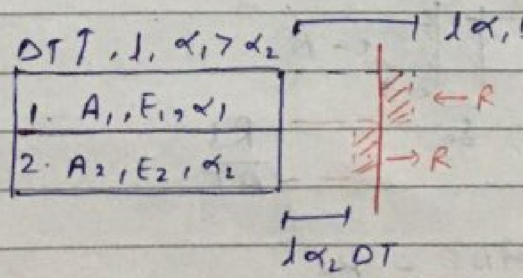


Total change in length = change in length due to temp + change in length due to loading

$$-\delta_0 = -l\alpha\Delta T + \frac{Rl}{AE}$$

$$R = \frac{AE}{l}(l\alpha\Delta T - \delta_0)$$

COMPOSITE BAR WITH TEMP CHANGE:



case of temp increase

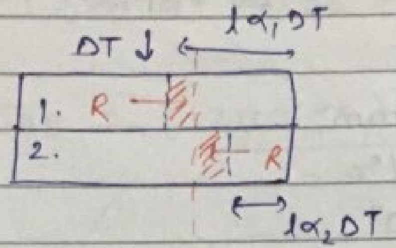
$$\frac{Rl}{A_1E_1} + \frac{Rl}{A_2E_2} = l(\alpha_1 - \alpha_2)\Delta T$$

Find R.

In case of temp ↑

Bar having larger α will have compression

Bar having small α will have tension



case of temp. Decrease

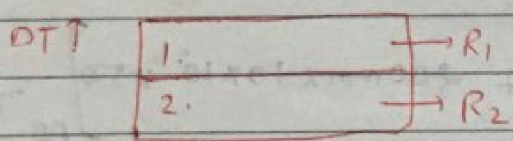
$$\frac{Rl}{A_1E_1} + \frac{Rl}{A_2E_2} = l(\alpha_1 - \alpha_2)\Delta T$$

find R. Magnitude.

In case of temp. Fall ↓

Bar having larger α will have tension

Bar having smaller α will have compression.



$$R_1 + R_2 = 0 \quad \text{-(I)}$$

$$\delta = l\alpha_1 \Delta T + \frac{R_1 l}{A_1E_1} = l\alpha_2 \Delta T + \frac{R_2 l}{A_2E_2} \quad \text{-(II)}$$

get R_1 \$ R_2

In case of temp fall the same relationship would be applied but ΔT will be taken as -ve.

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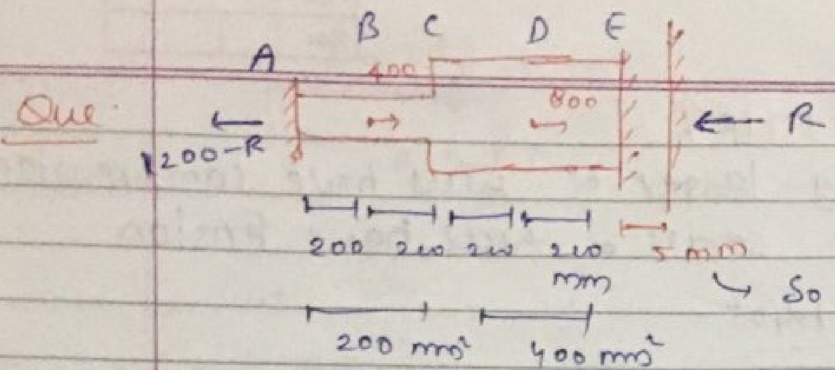

↳ So support yield

$E = 2\times10^5\ N/mm^2$

$\alpha = 10\times10^{-6}/^\circ C$

$\Delta T \uparrow = 50^\circ C$

$\xrightarrow{R}\square\xleftarrow{} R$

$800-R \leftarrow \square \rightarrow 800-R$

$800-R \leftarrow \square \rightarrow 800-R$

$1200-R \leftarrow \square \rightarrow 1200-R$

$$\delta_{AB} + \delta_{BC} + \delta_{CD} + \delta_{DE} = 5\,mm$$

$$5 = \left[\frac{(1200-R)\times10^3\times200}{200\times2\times10^5} + 200mm\times10\times10^{-6}\times50\right]_{AB} +$$

$$\left[\frac{(800-R)\times10^3\times200}{200\times2\times10^5} + 200\times10\times10^{-6}\times50\right]_{BC} +$$

$$\left[\frac{(800-R)\times10^3\times200}{400\times2\times10^5} + 200\times10\times10^{-6}\times50\right]_{CD} +$$

$$\left[\frac{-R\times10^3\times200}{400\times2\times10^5} + 200\times10\times10^{-6}\times50\right]_{DE}$$

$$5 = \left[\left(6-\frac{R}{200}\right)+0.1mm\right]_{AB} + \left[\left(4-\frac{R}{200}\right)+0.1\,mm\right]_{BC} + \left[\left(2-\frac{R}{400}\right)+0.1mm\right]_{CD}$$

$$+\left[-\frac{R}{400}+0.1\,mm\right]_{DE}$$

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$\Rightarrow R = 493.33\ kN$

elongation $\delta_{AB} = 3.633$ mm, $\delta_{BC} = 1.633$ mm elongation

elongation $\delta_{CD} = 0.866$ mm, $\delta_{DE} = -1.13$ mm contraction

$$\sigma_{AB} = \frac{(1200 - R)\times 10^3}{200}\ MPa = 3533.3\ MPa$$
(tensile)

$$\sigma_{BC} = \frac{(800 - R)\times 10^3}{200} = 1533.35\ MPa$$
tensile

$$\sigma_{CD} = \frac{(800 - R)\times 10^3}{400} = 766.67\ MPa$$
tensile.

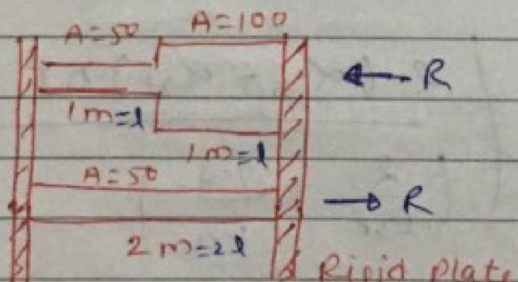
$$\sigma_{DE} = \frac{-R\times 10^3}{400} = -1233.325\ MPa$$
$\Rightarrow$ compressive.

NOTE In this case $\frac{\text{change in length of member}}{\text{original length of member}} \times E$

$\neq$ stress.

Because of the temp. effects.

Que.



$\Delta T\uparrow = 50°C$

$E_{cu} = 1\times10^5$, $\alpha_{cu} = 15\times10^{-6}/°C$

$E_{st} = 2\times10^5$, $\alpha_{st} = 12\times10^{-6}/°C$

Find stresses in bars?

In this case behaviour of st & cu will be composite Hence, change in length will be same

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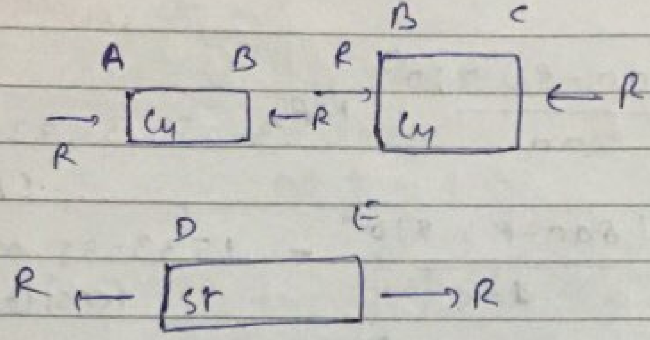
Since the free elongation in Cu will be greater then that of steel
Hence, copper will have compression and steel will have tension.



change in length in Cu = change in length in steel

$$\left[\left(l\alpha_{cu}\Delta T - \frac{RL}{AE_{cu}}\right) + \left(l\alpha_{cu}\Delta T - \frac{RL}{2AE_{cu}}\right)\right]_{cu} = \left[2l\alpha_{st}\Delta T + \frac{2Rl}{AE_{st}}\right]_{st}$$

$$2l\,\alpha_{cu}\Delta T - \frac{1.5\,RL}{AE_{cu}} = 2l\alpha_{st}\Delta T + \frac{2Rl}{AE_{st}}$$

$$2l(\alpha_{cu} - \alpha_{st})\Delta T = \frac{Rl}{A}\left(\frac{2}{E_{st}} + \frac{1.5}{E_{cu}}\right)$$

$$R = \left(\frac{2(\alpha_{cu} - \alpha_{st})\Delta T \cdot A}{\left(\frac{2}{E_{st}} + \frac{1.5}{E_{cu}}\right)}\right) N.$$

$R = 600\,N.$

$\sigma_{AB} = \frac{R}{A} = 12\ N/mm^2$ comp.

$\sigma_{BC} = \frac{R}{2A} = 6\ N/mm^2$ ~~tension~~ comp.

$\sigma_{DE} = R/A = 12\ N/mm^2$ tensile.

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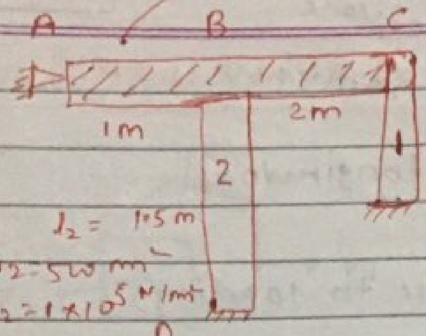
Que.



$1m = l_1$

$A_1 = 400\ mm^2$

$E_1 = 2\times10^5\ N/mm^2$

$\alpha_1 = 12\times10^{-6}/°C$

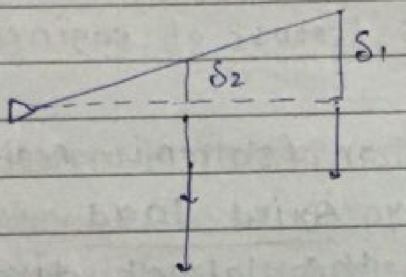
$l_2 = 1.5\ m$

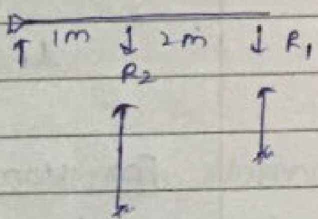
$A_2 = 500\ mm^2$

$E_2 = 1\times10^5\ N/mm^2$

$\alpha_2 = 10\times10^{-6}/°C$

$\Delta T \uparrow = 50°C$, Find stresses in 1 & 2





$$\delta_1 = l_1\alpha_1\Delta T + \frac{R_1 l_1}{A_1E_1}$$

$$\delta_2 = l_2\alpha_2\Delta T + \frac{R_2 l_2}{A_2E_2}$$

$$\Sigma M_A) = 0$$

$$R_1\times3 + R_2\times1 = 0$$

$$-3R_1 = R_2 \quad \text{—①}$$

$$\frac{\delta_1}{\delta_2} = \frac{3}{1}$$

$$\frac{l_1\alpha_1\Delta T + \frac{R_1l_1}{A_1E_1}}{l_2\alpha_2\Delta T + \frac{R_2l_2}{A_2E_2}} = 3$$

$$3\left(1.5\times10\times10^{-6}\times50 + \frac{R_2\times1.5}{500\times10^5\times10^{-6}}\right) = 1\times12\times10^{-6}\times50 + \frac{R_1\times1}{400\times2\times10^5\times10^{-6}}$$

$$\left(4.5\times10^{-4}\times5 - \frac{3R_1\times1.5}{50}\right) = 6\times10^{-4} + \frac{R_1}{80}$$

$$\left\{\sigma = R/A\right.$$

$$\frac{R_1}{80} + \frac{4.5R_1}{50} = 22.5\times10^{-4} - 6\times10^{-4}$$

$R_1 = 5.840\ kN$ $\quad \sigma_1 = 14.6\ N/m^2$

$R_2 = -17.522\ kN$ $\quad \sigma_2 = -35.4\ N/m^2$

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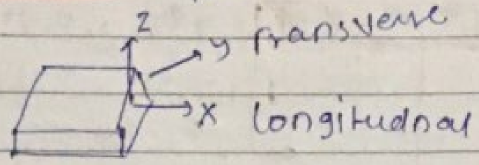
Poisson's Ratio :- (μ)

transverse



ε_x : (strain due to loading)

$\varepsilon_y = -\mu \varepsilon_x$

$\varepsilon_z = -\mu \varepsilon_z$

$\mu \rightarrow 0 - 0.5$ (Most of engineering materials)

For Homogeneous or Isotropic Material strain produced by any Axial load is accompnied by strains in the lateral directions and all such lateral strains are equal

The ratio of the lateral strain and longitudnal strain is called poisson's Ratio

$$\mu = -\frac{\text{lateral strain}}{\text{longitudnal strain}}$$

For most of the engineering materias poisson's Ratio is in b/w $\boxed{0 - 0.5}$. However the theortical Range is $(-1 \text{ to } 0.5)$

$M_{cork} = 0$

$M_{\text{perfectly elastic Rubber}} - 0.5$

Steel $\rightarrow 0.27 - 0.3$

Aluminium $\rightarrow 0.33$

cast Iron $\rightarrow 0.2 - 0.3$

Concrete $\rightarrow 0.1 - 0.2$

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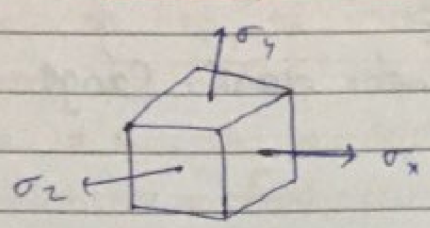
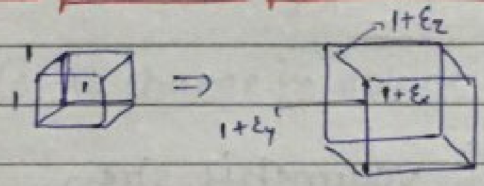
Multi-Axial loading :- (No shear stress)



$$\varepsilon_x = \sigma_x/E - \mu\sigma_y/E - \mu\sigma_z/E$$
$$\varepsilon_y = \sigma_y/E - \mu\sigma_x/E - \mu\sigma_z/E$$
$$\varepsilon_z = \sigma_z/E - \mu\sigma_x/E - \mu\sigma_y/E$$

NOTE even if shear stresses are acting Normal strains will be given by above formula if the material is Homogeneous & Isotropic that is shear stresses don't affect Normal strain if the material is Homogeneous & Isotropic

Dilation & Bulk Modulus :-



$$\varepsilon_x = \frac{\Delta l_x}{1} = \Delta l_x$$

$$\frac{\Delta V}{V} = \varepsilon_v = \text{volumetric change}$$

$$\varepsilon_v = \frac{(1+\varepsilon_x)(1+\varepsilon_y)(1+\varepsilon_z) - 1}{1}$$

Neglected for small strain

$$\varepsilon_v = (\varepsilon_x + \varepsilon_y + \varepsilon_z)(\varepsilon_x\varepsilon_y + \varepsilon_y\varepsilon_z + \varepsilon_x\varepsilon_z + \varepsilon_x\varepsilon_y\varepsilon_z)$$

$$\boxed{\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z}$$

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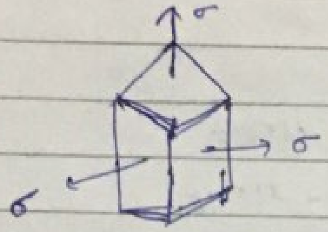
$$\boxed{\varepsilon_v = \frac{(\sigma_x + \sigma_y + \sigma_z)(1-2\mu)}{E}}$$

valid only in linear elastic Range



σ, volumetric stress

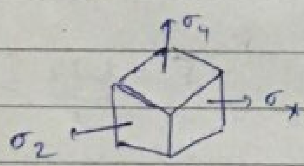
$$\varepsilon_v = \frac{3\sigma(1-2\mu)}{E}$$

↓
volumetric strain

$$\frac{\sigma}{\varepsilon_v} = \text{Bulk modulus } (K)$$

$$\boxed{K = \frac{E}{3(1-2\mu)}}$$

$1/K \rightarrow$ compressibility.



volumetric stress $= \frac{\sigma_x + \sigma_y + \sigma_z}{3}$

$$\varepsilon_v = \frac{(\sigma_x + \sigma_y + \sigma_z)/3}{E/3(1-2\mu)}$$

NOTE For Engineering materials Range of Bulk modulus is $\boxed{E/3 \text{ to } \infty}$

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Que. A steel cube of side a is contained in a rigid cubical box of inside length 'a'. Box is closed from all sides and temp of the cube is raised by $t°C$, its α = coff. thermal Expension, $E \to$ Modulus of Elasticity, find the stresses developed on the sides.

Total change in length

$$\text{in x-dir}^n = \underbrace{a\alpha t}_{\text{change in length due to temp}} + \underbrace{\epsilon_x \cdot a}_{\text{due to loading}} = 0$$

$$a\alpha t + \left(-\frac{\sigma}{E} - \frac{\mu(-\sigma)}{E} - \frac{\mu(-\sigma)}{E}\right) a = 0$$

$$a\alpha t - \sigma/_E (1-2\mu) a = 0$$

$$\boxed{\sigma = \frac{E\alpha t}{1-2\mu}}$$

Alternativ.

Total change in volume = volume change due to temp. + vol. change due to stress = 0

$$= \left(3\frac{a\alpha t}{a}\right) a^3 + \frac{(-\sigma-\sigma-\sigma)(1-2\mu)}{E} \cdot a^3 = 0$$

$$\alpha t - \frac{\sigma(1-2\mu)}{E} = 0$$

$$\sigma = \frac{E\alpha t}{1-2\mu}.$$

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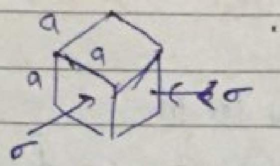
Que. In the above problem if top cover is removed and temp. is raised by $t°C$ Find the stresses on the sides of the cube and the elongation in vertical dir^n.



Total change in length

$$\text{in } x\text{-dir}^n = \underbrace{a\alpha t}_{\text{due to temp.}} + \underbrace{\epsilon_x a}_{\text{due to stress}} = 0$$

$$a\alpha t + \left(\frac{-\sigma}{E} - \frac{\mu(-\sigma)}{E}\right) \cdot a = 0$$

$$a\alpha t - \frac{\sigma}{E}(1-\mu)a = 0$$

$$\boxed{\sigma = \frac{E\alpha t}{1-\mu}}$$

$$\Delta l_y = a\alpha t + \epsilon_y \cdot a$$

$$= a\alpha t + \left(\frac{0}{E} - \frac{\mu(-\sigma)}{E} - \frac{\mu(-\sigma)}{E}\right) a$$

$$= a\alpha t + \frac{2\mu a}{E}(\sigma)$$

$$= a\alpha t + \frac{2\mu a}{E}\left(\frac{E\alpha t}{1-\mu}\right)$$

$$= a\alpha t\left[1 + \frac{2\mu}{1-\mu}\right]$$

$$= a\alpha t\,\frac{(1+\mu)}{(1-\mu)}$$

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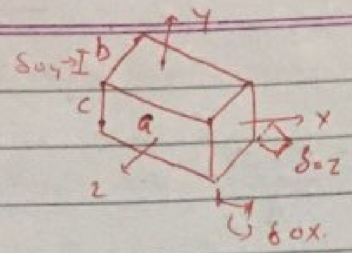
Que.



$$a\alpha t + \varepsilon_x \cdot a = \delta_{ox} \quad -①$$
$$c\alpha t + \varepsilon_y c = \delta_{oy} \quad -②$$
$$b\alpha t + \varepsilon_z b = \delta_{oz} \quad -③$$

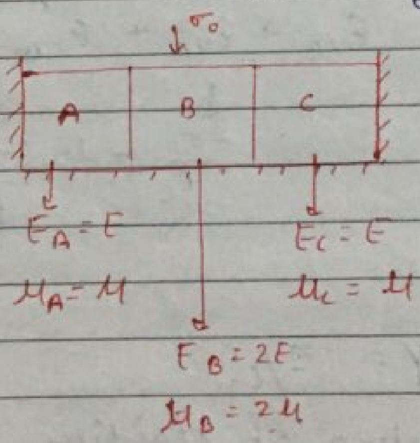
$$\varepsilon_x = \frac{-\sigma_1}{E} - \frac{\mu(-\sigma_2)}{E} - \frac{\mu(-\sigma_3)}{E}$$

$$\varepsilon_y = \frac{-\sigma_2}{E} - \frac{\mu(-\sigma_3)}{E} - \frac{\mu(-\sigma_1)}{E}$$

$$\varepsilon_z = \frac{-\sigma_3}{E} - \frac{\mu(-\sigma_1)}{E} - \frac{\mu(-\sigma_2)}{E}$$

solve it.

Que.

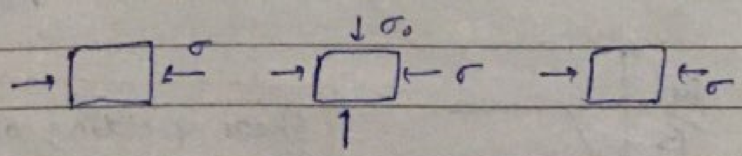


sides of all cubes are a.

find the stresses at the interfaces of the cube.

$$\Delta l_{xA} + \Delta l_{xB} + \Delta l_{xC} = 0$$
$$\varepsilon_{xA} a + \varepsilon_{xB} a + \varepsilon_{xC} a = 0$$

$$\varepsilon_{xA} + \varepsilon_{xB} + \varepsilon_{xC} = 0$$



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$\varepsilon_{xA} = -\sigma / E_A$

$\varepsilon_{xB} = \dfrac{-\sigma}{E_B} - \mu_B \left(- \dfrac{\sigma_0}{E_B} \right)$

$\varepsilon_{xC} = -\sigma / E_C$

$\Rightarrow \left(\dfrac{-\sigma}{E} \right) + \left(\dfrac{-\sigma}{2E} - \dfrac{(2\mu)(-\sigma_0)}{2E} \right) + \left(\dfrac{-\sigma}{E} \right) = 0$

$= -2.5 \dfrac{\sigma}{E} + \dfrac{\mu \sigma_0}{E} = 0$

$$\boxed{\sigma = \frac{\mu \sigma_0}{2.5}}$$

Volume change in B.

$\dfrac{0}{E_B} - \mu_B \left(\dfrac{-\sigma_0}{E_B} \right) - \mu_B \left(\dfrac{-\sigma}{E_B} \right)$

$\dfrac{\Delta V_B}{V_B} = \dfrac{\Delta V_B}{a^3} = \varepsilon_{xB} + \varepsilon_{yB} + \varepsilon_{zB}$

$\varepsilon_{yB} = \dfrac{-\sigma_0}{E_B} - \mu_B \dfrac{(-\sigma)}{E_B}$

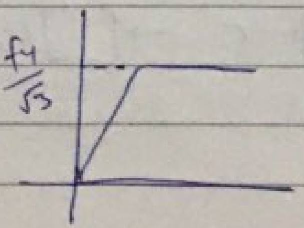
Shearing strain:-

$\gamma = \tau / G$

$G \rightarrow$ Modulus of rigidity

$$\boxed{G = \frac{E}{2(1+\mu)}}$$

Range of $G \rightarrow E/2$ to $E/3$ for engineering materials.



Shear yielding occurs at $f_y / \sqrt{3}$

For Homogeneous & Isotropic Material

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\mu\sigma_y}{E} - \frac{\mu\sigma_z}{E}$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \frac{\mu\sigma_x}{E} - \frac{\mu\sigma_z}{E}$$

$$\varepsilon_z = \frac{\sigma_z}{E} - \frac{\mu\sigma_x}{E} - \frac{\mu\sigma_y}{E}$$

$$\gamma_{xy} = \tau_{xy}/G$$

$$\gamma_{yz} = \tau_{yz}/G$$

$$\gamma_{zx} = \tau_{zx}/G$$

Generalised Hook's law.

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{bmatrix} = \underbrace{\begin{bmatrix} 1/E & -\mu/E & -\mu/E & 0 & 0 & 0 \\ -\mu/E & 1/E & -\mu/E & 0 & 0 & 0 \\ -\mu/E & -\mu/E & 1/E & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/G & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G \end{bmatrix}}_{\Downarrow} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{bmatrix}$$

elastic constant matrix

square symmetrical matrix for all type of matrix

Number of independent elastic constant in elastic constant matrix for Homogeneous and Isotropic material is 2.

→ Number of independent elastic constants in elastic constant matrix for anisotropic material is $\boxed{21}$

* → Number of independent elastic constants in elastic constant matrix for orthotropic material is $\boxed{9}$

MATHEMATICAL DEFINATIONS OF Strain :-

if u, v, w are deforms in x, y, z dir^n.

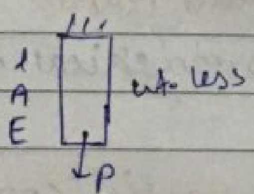
$$\varepsilon_x = \partial u/\partial x \quad , \quad \gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$

$$\varepsilon_y = \partial v/\partial y \quad \quad \gamma_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}$$

$$\varepsilon_z = \partial w/\partial z \quad \quad \gamma_{zx} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}$$

Strain Energy due to Axial loading :-

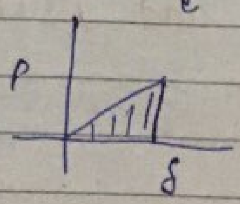
strain energy per unit volume $= \frac{1}{2} . \sigma . \varepsilon$



Total strain energy $= \frac{1}{2} \sigma . \varepsilon \times A \times l$

- Area under load deformation curve - Total strain energy store.

$= \frac{1}{2} P \delta$



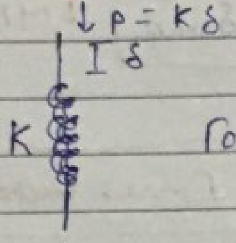
- Area under $\sigma - \varepsilon$ Curve is total strain energy per unit volume

Total strain energy $= \frac{1}{2} \times P \times \delta$

$$= \frac{1}{2} \times P \times \frac{Pl}{AE}$$

$$= \frac{P^2 l}{2AE}$$

For spring



Total strain energy $= \frac{1}{2} \times P \times \delta$

$$= \frac{1}{2} \times P \times P/K = \frac{P^2}{2K}$$

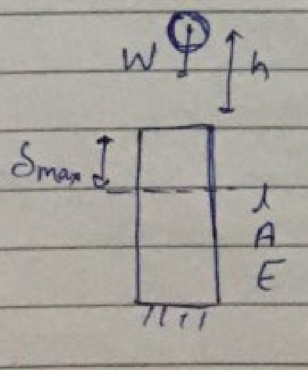
$$= \frac{1}{2}(K\delta)\delta$$

$$= \frac{1}{2} K\delta^2$$

$$\boxed{\text{Total strain energy} = \int \frac{P^2\,dx}{2AE}}$$

→ principle of superposition can't be applied in case of strain energy calculation

IMPACT loading :-



$$W(h+\delta_{max}) = \frac{1}{2}\frac{AE}{l}\cdot\delta_{max}^2$$

$$\delta_{max}^2 - \left(\frac{2Wl}{AE}\right)\delta_{max} - \frac{2Wlh}{AE} = 0$$

Solve Quadratic

$$\delta_{max} = \frac{\frac{2Wl}{AE} + \sqrt{\left(\frac{2Wl}{AE}\right)^2 + \frac{8Wlh}{AE}}}{2}$$

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$$\delta_{static} = \frac{Wl}{AE}$$

$$\delta_{max.} = \delta_{static} + \delta_{static}\sqrt{1 + \frac{2h}{\delta_{static}}}$$

$$\boxed{\frac{\delta_{max}}{\delta_{static}} = 1 + \sqrt{1 + \frac{2h}{\delta_{static}}}}$$ Impact factor

$\boxed{\text{Dynamic loading} \times \text{Impact factor} = \text{static loading}}$

Equivelent static load, $P_{dyn} = W \times IF$

$$\delta_{max} = \frac{P_{dyn} \times l}{AE}$$

$$\sigma_{max} = \frac{P_{dyn}}{A}$$

NOTE: if $h = 0$, then the loading can be treated as sudden loading and in this case impact factor $= 2$, Hence

$$\delta_{max.} = 2\,\delta_{static}$$

and $\sigma_{max} = 2\,W/A = 2\,\sigma_{static}$

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